الجامعة السورية الخاصة
SYRIAN PRIVATE UNIVERSITY

كـلـيــة هندسة الحاسوب والمعلوماتية
Computer and Informatics Engineering Faculty

Electric Circuits I

Dr. Eng.
Hassan M. Ahmad
Hassan.Ahmad@spu.edu.sy, istamo48@mail.ru

Chapter 5
Operational Amplifier

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5.2 Ideal Op Amp
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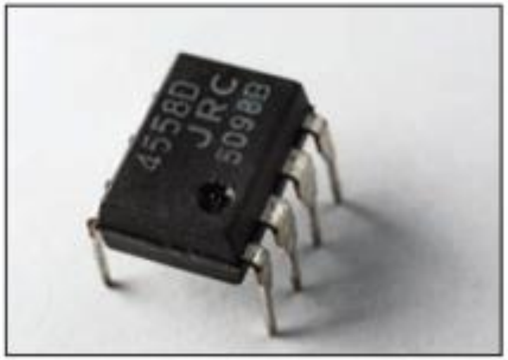
5.1 What is an Op Amp

- The **operational amplifier** , or (**op amp** for short) is an electronic unit that behaves like a voltage-controlled voltage source (VCVS).

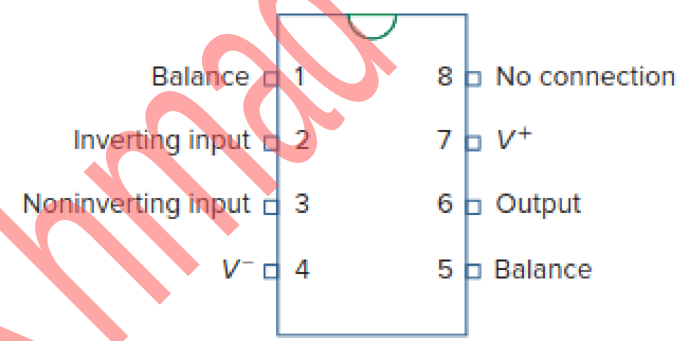
- It is an active circuit element designed to perform mathematical operations of ***addition***, ***subtraction***, ***multiplication***, ***division***, ***differentiation*** and ***integration***.

- Op amps are *commercially* available in *integrated circuit* (IC) *packages* in several forms as shown in Fig. for a typical operational amplifier.



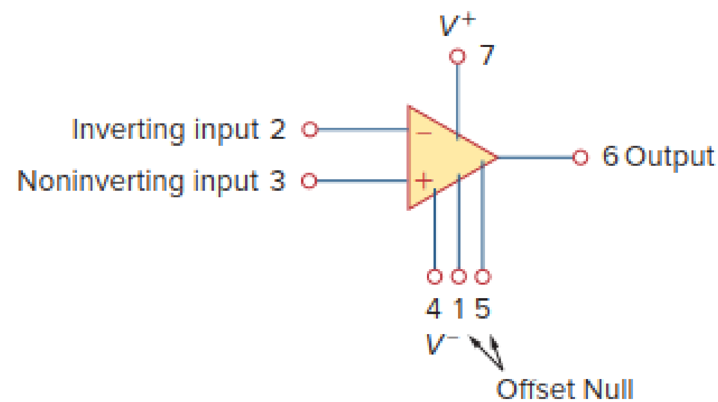
Op Amp

- A typical Op amp is the eight-pin dual in-line package (or DIP), Fig. (a).
- Pin or terminal 8 is unused, and terminals 1 and 5 are of little concern to us.
- The **five important terminals** are:
  - The inverting input, pin 2.
  - The noninverting input, pin 3.
  - The output, pin 6.
  - The positive power supply V^+ , pin 7.
  - The negative power supply V^- , pin 4.



(a)

- The **circuit symbol** for the op amp is the triangle in Fig.(b);
  - The inputs are marked with minus (−) and plus (+) to specify *inverting* and *noninverting* inputs.

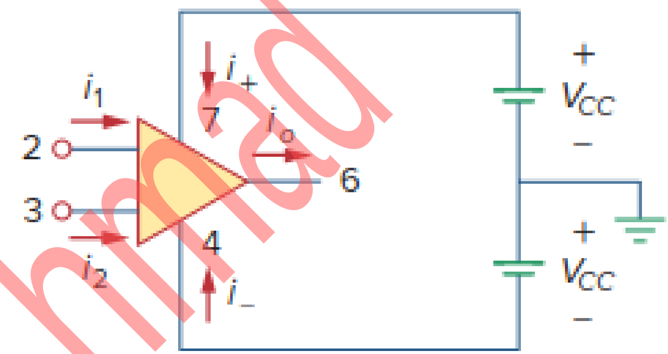


(b)

Op Amp

- As an active element, the **op amp** must be powered by a voltage supply (V_{CC}), Fig. (a)
  - By applying KCL:

$$i_o = i_1 + i_2 + i_+ + i_-$$

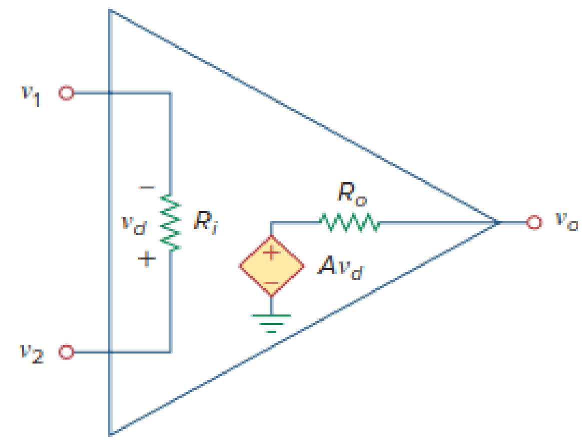


(a)

- The **equivalent circuit model** of an op amp, Fig.(b).
  - R_i is the Thevenin equivalent resistance seen at the input terminals, while R_o is the Thevenin equivalent resistance seen at the output.
  - The differential input voltage v_d is given by

$$v_d = v_2 - v_1$$

where v_1 – inverting terminal voltage; v_2 – noninverting terminal voltage.



(b)

- The output v_o is given by $v_o = Av_d = A(v_2 - v_1)$

where A is called the **open-loop voltage gain** because it is the gain of the op amp *without* any *external feedback* from output to input.

- When there is a feedback path from output to input, the ratio of the output voltage to the input voltage is called the **closed-loop gain**.

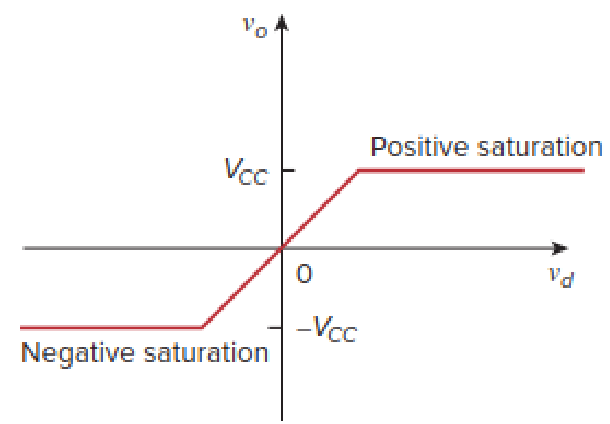
Op Amp

- **Typical ranges for op amp parameters** (see table).

Parameter	Typical range	Ideal values
Open-loop gain, A	10^5 to 10^8 Ω	∞ Ω
Input resistance, R_i	10^5 to 10^{13} Ω	∞ Ω
Output resistance, R_o	10 to 100 Ω	0 Ω
Supply voltage, V_{CC}	5 to 24 V	

- The op amp can operate in **three modes**, depending on the differential input voltage v_d, as shown in Fig.
  1. Positive saturation, $v_o = V_{CC}$.
  2. Linear region ,

     $-V_{CC} \le v_o = Av_d \le V_{CC}$.
  3. Negative saturation, $v_o = -V_{CC}$.



- We will assume that our op amps operate in the linear mode, the output voltage is restricted by

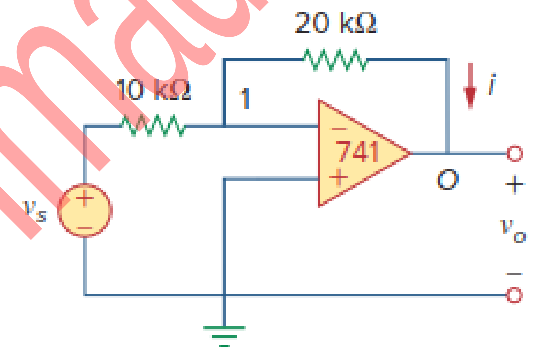
$$-V_{CC} \le v_o \le V_{CC}$$

Example 5.1

A 741 op amp has an open-loop voltage gain of 2×10^5, input resistance of 2 MΩ, and output resistance of 50 Ω. The op amp is used in the circuit of Fig.(a).

a) Find the closed-loop gain v_o/v_s.

b) Determine current i when v_s = 2 V.



(a)

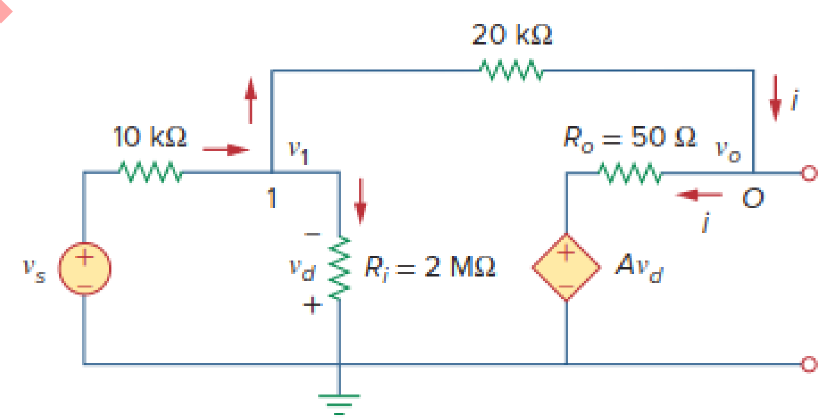
Solution

Using the equivalent circuit model for op amp, we obtain the equivalent circuit of Fig.(a) as shown in Fig.(b).

At **node 1**, KCL gives

$$\frac{v_s - v_1}{10\times10^3} = \frac{v_1}{2000\times10^3} + \frac{v_1 - v_o}{20\times10^3} \Rightarrow 200v_s = 301v_1 - 100v_o \Rightarrow 2v_s \simeq 3v_1 - v_o$$

$$\Rightarrow \qquad v_1 = \frac{2v_s + v_o}{3} \quad (1)$$



(b)

At **node *O***, $\dfrac{v_1 - v_o}{20\times10^3} = \dfrac{v_o - Av_d}{50}$, but $v_d = v_1$

$$\Rightarrow \quad v_1 - v_o = 400[v_o + (2\times10^5)v_1] \quad (2)$$

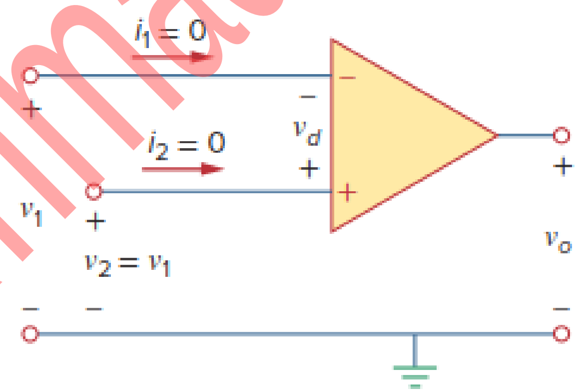
Substituting v_1 from Eq. (1) into Eq. (2) gives

$$0 \simeq 26667067v_o + 53333333v_s \Rightarrow \frac{v_o}{v_s} = -1.9999699$$

$$v_s = 2V \Rightarrow v_o = -3.9999398V \text{ and } v_1 = 20.066667V \Rightarrow i = \frac{v_1 - v_o}{20\times10^3} = 0.19999\,\text{mA}$$

5.2 Ideal Op Amp

- An ideal op amp has the following characteristics:
  - Infinite open-loop gain, $A \approx \infty$
  - Infinite input resistance, $R_i \approx \infty$
  - Zero output resistance, $R_o \approx 0$.



- **Two important characteristics** of the ideal op amp are:
  1. The currents into both input terminals are zero:

$$i_1 = 0, \quad i_2 = 0$$

  2. The voltage across the input terminals is equal to zero; i.e.,

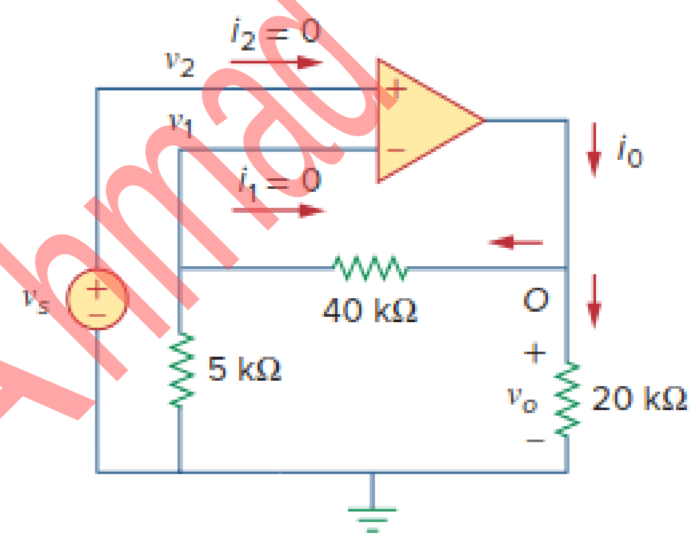
$$v_d = v_2 - v_1 = 0 \quad \Rightarrow v_1 = v_2$$

Example 5.2.

Using the ideal op amp model in Fig.

a) Find the closed-loop gain v_o/v_s.

b) Determine current i_o when v_s = 1 V.



Solution:

- Notice that $v_2 = v_s$
- Since $i_1 = 0$, the 40-kΩ and 5-kΩ resistors are in series; The same current flows through them.
- v_1 is the voltage across the 5-kΩ resistor.
- Hence, using the voltage division principle (VDR),

$$v_1 = \frac{5}{5+40} v_o = \frac{v_o}{9}$$

- For ideal op amp, we know that $v_2 = v_1$

So, $$v_2 = v_1 = v_s = \frac{v_o}{9} \Rightarrow \frac{v_o}{v_s} = 9$$

- At node *O*, (KCL):

$$i_o = \frac{v_o}{40+5} + \frac{v_o}{20} \ \text{mA}$$

- When v_s =1 V ➔ v_o = 9 V.

Then,

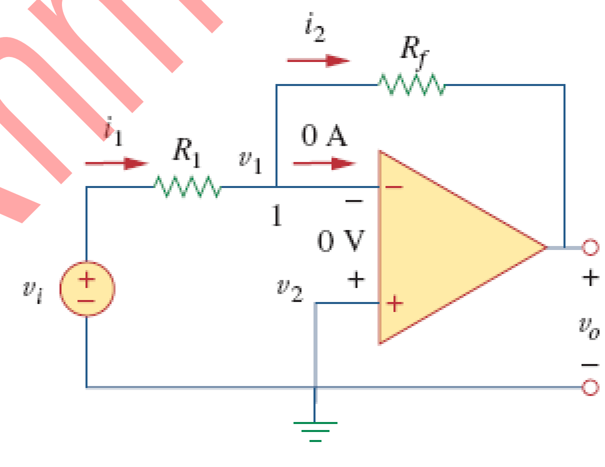
$$i_o = 0.2 + 0.45 = 0.65 \ \text{mA}$$

5.3 Configuration of Op amp

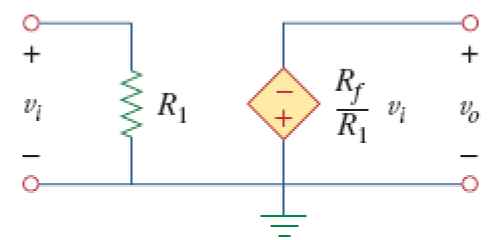
1. **Inverting amplifier** reverses the polarity of the input signal while amplifying it.

- The circuit of inverting amplifier is shown in Fig.(a).
  - The noninverting input is grounded,
  - v_i is connected to the inverting input through R_1,
  - The feedback resistor R_f is connected between the inverting input and output.
- **Our goal** is to obtain the relationship between the *input voltage* v_i and the *output voltage* v_o.
  - Applying KCL at node 1, $i_1 = i_2 \Rightarrow \dfrac{v_i - v_1}{R_1} = \dfrac{v_1 - v_o}{R_f}$
  - But $v_1 = v_2 = 0$ for an ideal op amp, since the noninverting terminal is grounded. Hence,

$$\frac{v_i}{R_1} = -\frac{v_o}{R_f} \Rightarrow v_o = -\frac{R_f}{R_1} v_i$$



(a)



(b)

- The **voltage gain** is

$$A_v = \frac{v_o}{v_i} = -\frac{R_f}{R_1}$$

- The equivalent circuit for the inverting amplifier is shown in Fig. (b)

Example 5.3.

Refer to the op amp in Fig. If $v_i = 0.5V$, calculate:

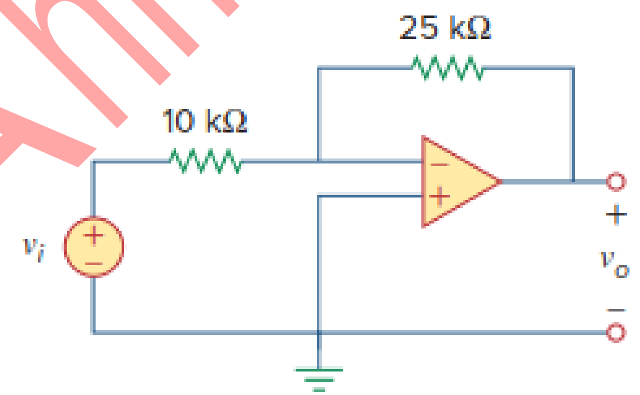
(a) the output voltage, v_o ;

(b) the current in the 10kΩ resistor.

Solution:

a) Output voltage:

$$v_o = -\frac{R_f}{R_1} v_i = -\frac{25}{10}(0.5) = -1.25 \text{ V}$$



b) the current through the 10kΩ resistor is:

$$i = \frac{v_i - 0}{R_1} = \frac{0.5 - 0}{10 \times 10^3} = 50 \mu A$$

Example 5.4.

Determine v_o in the op amp circuit shown in Fig.

Solution:

Applying KCL at node *a*,

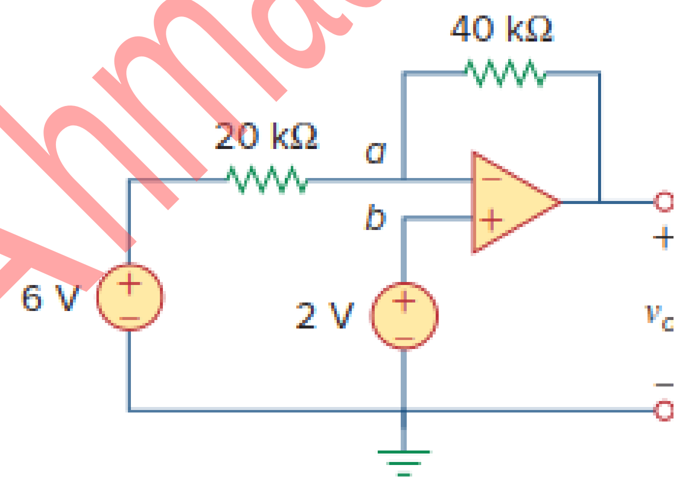
$$\frac{v_a - v_o}{40\,\text{k}\Omega} = \frac{6 - v_a}{20\,\text{k}\Omega} \Rightarrow v_a - v_o = 12 - 2v_a$$

$$\Rightarrow v_o = 3v_a - 12$$

But $v_a = v_b = 2$ V for an ideal op amp. Hence,

$$v_o = 3 \times 2 - 12 = -6\,\text{V}$$

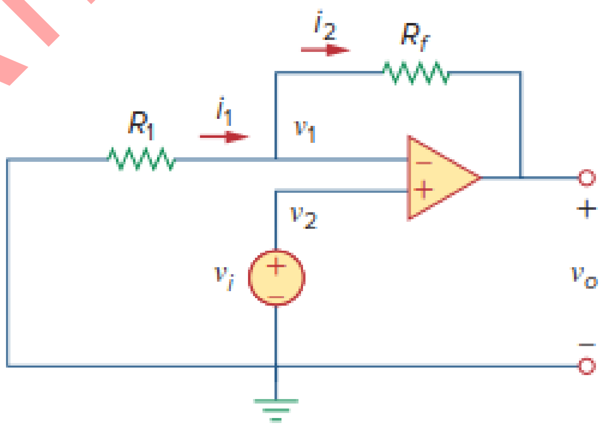
Notice that if $v_b = 0 = v_a$, then $v_o = -12$.



2. **Non-inverting amplifier** is designed to produce positive voltage gain.

- The circuit of the op amp is the noninverting amplifier is shown in Fig.
  - Application of KCL at the inverting terminal gives

$$i_1 = i_2 \Rightarrow \frac{0 - v_1}{R_1} = \frac{v_1 - v_o}{R_f}$$



- But $v_1 = v_2 = v_i$,

so

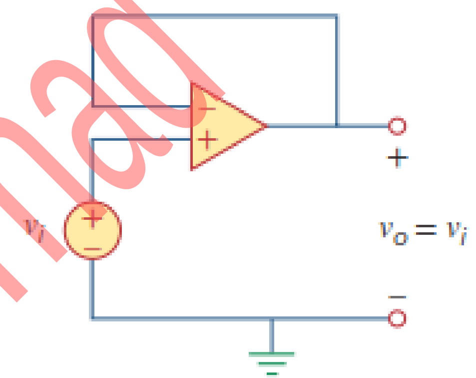
$$\frac{-v_i}{R_1} = \frac{v_i - v_o}{R_f} \Rightarrow \quad v_o = \left(1 + \frac{R_f}{R_1}\right) v_i$$

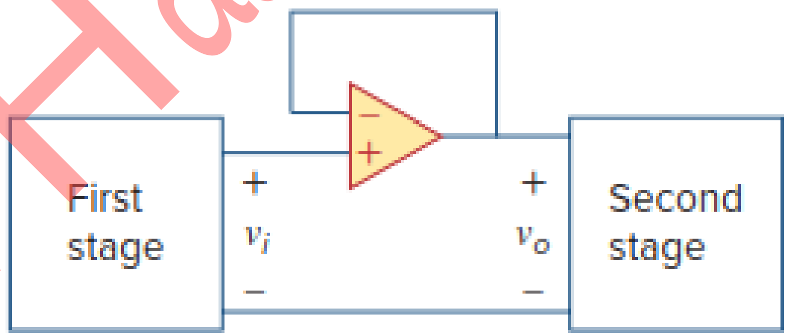
- The **voltage gain** is $A_v = \dfrac{v_o}{v_i} = 1 + \dfrac{R_f}{R_1}$

which does not have a negative sign.

Thus, the output has the same polarity as the input.

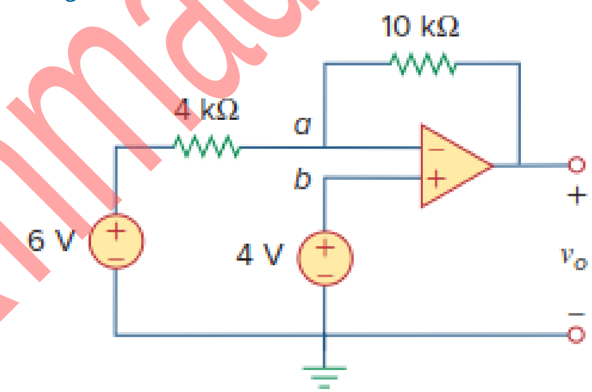
- In noninverting amplifier circuit, if feedback resistor $R_f = 0$ (short circuit) or $R_1 = \infty$ (open circuit) or both, the gain becomes 1.
  - Under these conditions, the circuit is called a **voltage follower (تابع/متتبع الجهد)** (or *unity gain amplifier*). Hence, $v_o = v_i$
  - Such a circuit has a very high input impedance and is therefore useful as an intermediate-stage (or **buffer**) amplifier (مكبر صَادّ للتغذية المرتدة) to isolate one circuit from another, as in Fig.





Example 5.5.

For the op amp shown in Fig., calculate the output voltage v_o.



Solution:

METHOD 1. Using superposition, we let

$$v_o = v_{o1} + v_{o2}$$

where v_{01} is due to the 6-V voltage source, and v_{02} is due to the 4-V input.

- To get v_{o1}, we set the 4-V source equal to zero. The circuit becomes an invert. amp.

$$v_{o1} = -\frac{10}{4}(6) = -15 \text{ V}$$

- To get v_{02}, we set the 6-V source equal to zero. The circuit becomes a noninvert. amp. , so that

$$v_{o2} = \left(1 + \frac{10}{4}\right)4 = 14 \text{ V}$$

- Thus, $v_o = v_{o1} + v_{o2} = -15 + 14 = -1 \text{ V}$

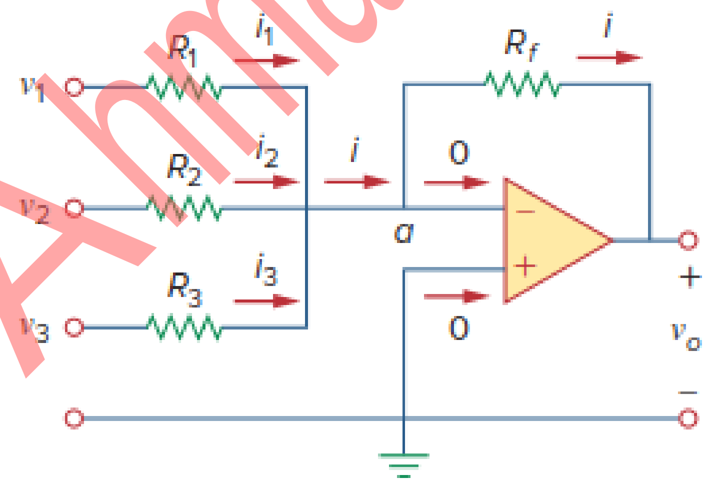
METHOD 2. Using nodal analysis.

- Applying KCL at node ***a***,

$$\frac{6 - v_a}{4} = \frac{v_a - v_o}{10}$$

- But $v_a = v_b = 4$, so $5 = 4 - v_o \Rightarrow v_o = -1 \text{ V}$

3. Summing Amplifier is an op amp circuit that combines several inputs and produces an output that is the weighted sum of the inputs.



- Applying KCL at node ***a*** gives:

$$i = i_1 + i_2 + i_3$$

- But

$$i_1 = \frac{v_1 - v_a}{R_1}; \; i_2 = \frac{v_2 - v_a}{R_2}$$

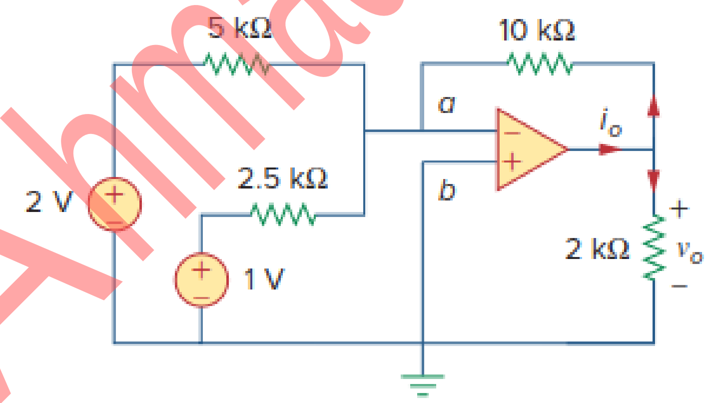
$$i_3 = \frac{v_3 - v_a}{R_3}; \; i = \frac{v_a - v_o}{R_f}$$

- We note that $v_a = 0$, thus

$$v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3 \right)$$

Example 5.6.

Calculate v_o and i_o in the op amp circuit shown below.



Solution:

This is a summer with two inputs.

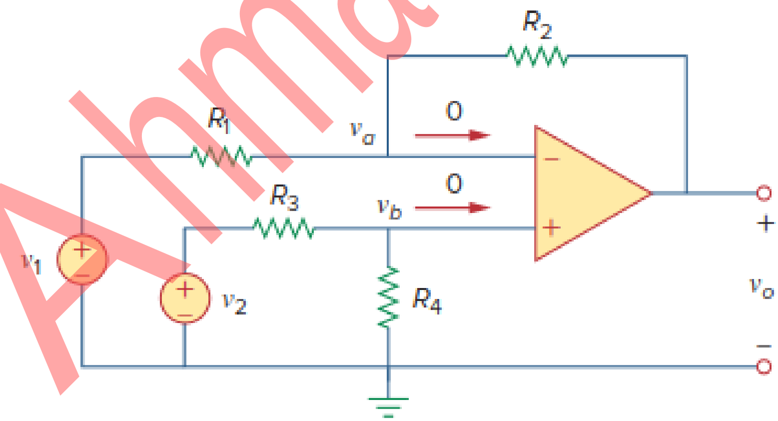
$$v_o = -\left[\frac{10}{5}(2) + \frac{10}{2.5}(1)\right] = -(4+4) = -8\,\text{V}$$

- The current i_o is the sum of the currents through the 10-kΩ and 2-kΩ resistors.
- Both of these resistors have voltage $v_o = -8$ V across them,

  since $v_a = v_b = 0$.
- Hence,

$$i_0 = \frac{v_o - 0}{10} + \frac{v_o - 0}{2} = -0.8 - 4 = -4.8\ \text{mA}$$

4. Difference amplifier is a device that amplifies the difference between two inputs but rejects any signals common to the two inputs.



- Applying KCL to node a,

$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2} \Rightarrow v_o = \left(\frac{R_2}{R_1} + 1\right) v_a - \frac{R_2}{R_1} v_1$$

- Applying KCL to node b,

$$\frac{v_2 - v_b}{R_3} = \frac{v_b - 0}{R_4} \Rightarrow v_b = \frac{R_4}{R_3 + R_4} v_2$$

- But $v_a = v_b$, thus

or

$$v_o = \left(\frac{R_2}{R_1} + 1\right) \frac{R_4}{R_3 + R_4} v_2 - \frac{R_2}{R_1} v_1$$

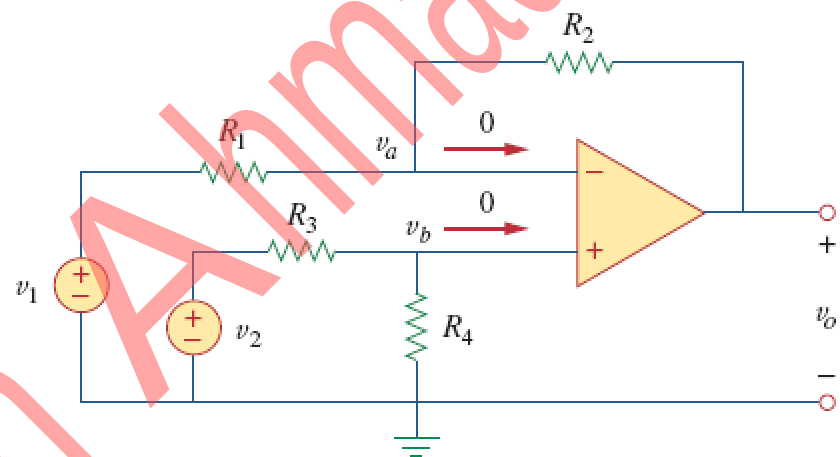
$$v_o = \frac{R_2 (1 + R_1 / R_2)}{R_1 (1 + R_3 / R_4)} v_2 - \frac{R_2}{R_1} v_1$$

The difference amplifier becomes a **subtractor**, with the output

$$\text{if } \frac{R_2}{R_1} = \frac{R_3}{R_4} = 1 \Rightarrow v_o = v_2 - v_1$$

Example 5.7.

Determine R_1, R_2, R_3 and R_4 so that $v_o = -5v_1 + 3v_2$ for the circuit shown below.



Solution:

- The output for this amplifier is

$$v_o = \frac{R_2(1 + R_1/R_2)}{R_1(1 + R_3/R_4)} v_2 - \frac{R_2}{R_1} v_1$$

- Comparing with

$$v_o = 3v_2 - 5v_1$$

we see that

$$\frac{R_2}{R_1} = 5 \Rightarrow R_2 = 5R_1$$

Also,

$$5\frac{(1 + R_1/R_2)}{(1 + R_3/R_4)} = 3 \Rightarrow \frac{\frac{6}{5}}{(1 + R_3/R_4)} = \frac{3}{5}$$

Or

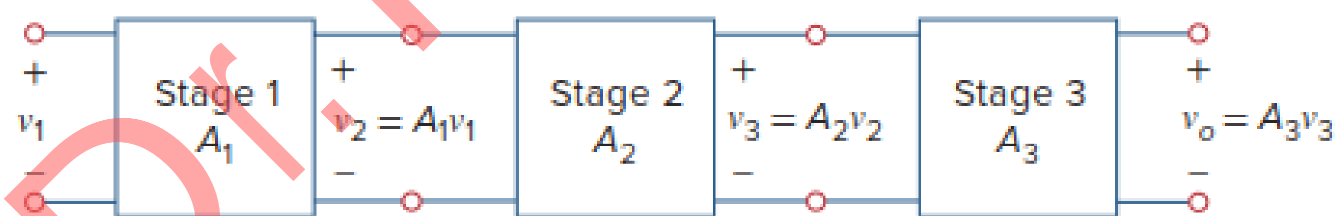
$$2 = 1 + \frac{R_3}{R_4} \Rightarrow R_3 = R_4$$

If we choose $R_1 = 10\text{k}\Omega$ and $R_3 = 20\text{k}\Omega$, then $R_2 = 50\text{k}\Omega$ and $R_4 = 20\text{k}\Omega$.

5.4 Cascaded Op Amp

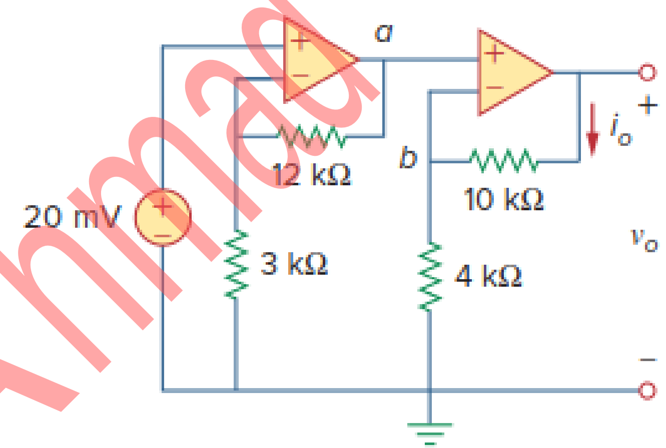
- A cascade connection is a head-to-tail arrangement of two or more op amp circuits such that the output to one is the input of the next.
  - When op amp circuits are cascaded, each circuit in the string is called a *stage*.
  - The original input signal is *increased* by the *gain* of the individual stage.
  - Figure displays a block diagram representation of three op amp circuits in cascade.
  - Since the output of one stage is the input to the next stage, the overall gain of the cascade connection is the product of the gains of the individual op amp circuits, or

$$A = A_1 A_2 A_3$$



Example 5.8.

Find v_o and i_o in the circuit shown in Fig.



Solution:

This circuit consists of two noninverting amplifiers cascaded.

- At the output of the first op amp,

$$v_a = \left(1 + \frac{12}{3}\right)(20) = 100 \text{ mV}$$

- At the output of the second op amp, $$v_o = \left(1 + \frac{10}{4}\right) v_a = 350 \text{ mV}$$

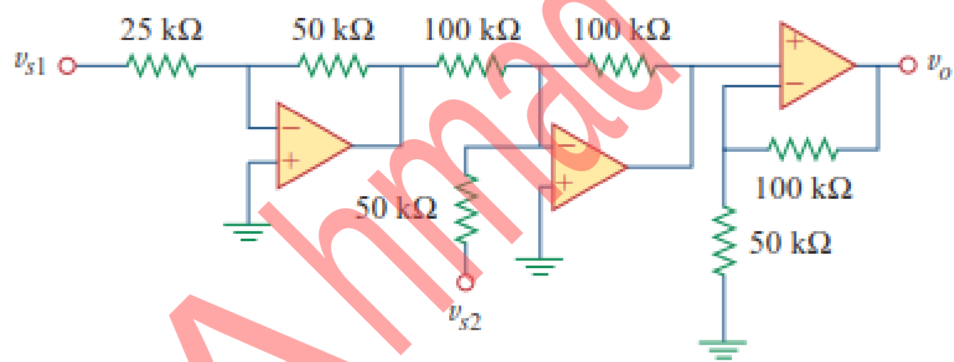
- The required current i_o is the current through the 10-kΩ resistor: $$i_o = \frac{v_o - v_b}{10} \text{ mA}$$

- But $v_b = v_a = 100 \text{ mV}$

- Hence, $$i_o = \frac{(350 - 100) \times 10^{-3}}{10 \times 10^3} = 25 \mu\text{A}$$

Example 5.9.

Find v_o in the op amp circuit of Fig.



Solution:

- Let v_1 output of the first op amp and v_2 output of the second op amp.
- The *first stage* is an inverting amplifier: $v_1 = -\frac{50}{25} v_{s1}$
- The *second state* is a summer:

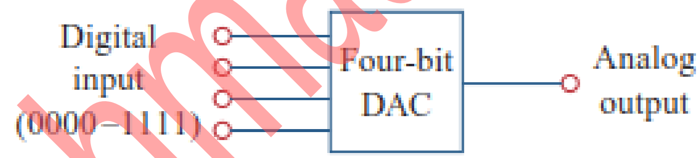
$$v_2 = -\left(\frac{100}{50} v_{s2} + \frac{100}{100} v_1\right) = -2v_{s2} - v_1 = -2v_{s2} + 2v_{s1}$$

- The *third state* is a noninverting amplifier:

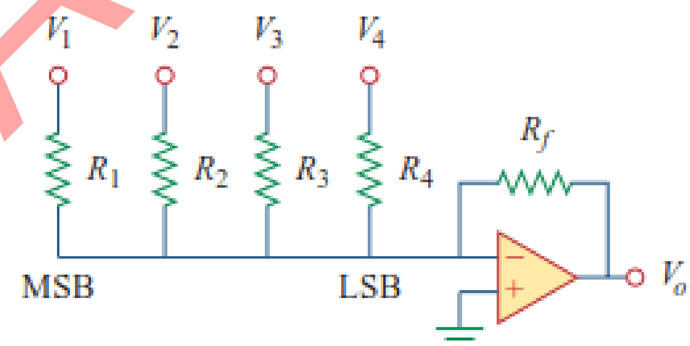
$$v_o = \left(1 + \frac{100}{50}\right) v_2 = 3v_2 = 3(-2v_{s2} + 2v_{s1}) = 6v_{s1} - 6v_{s2}$$

5.5 Application: Digital-to Analog Converter

- The **digital-to-analog converter (DAC)** transforms digital signals into analog form.
  - A typical example of a four-bit DAC is illustrated in Fig.(a).
  - The **four-bit DAC** can be realized in many ways.
  - A simple realization is the **binary weighted ladder (سلم = درج موزون ثنائي)**, Fig. (b).



(a)



(b)

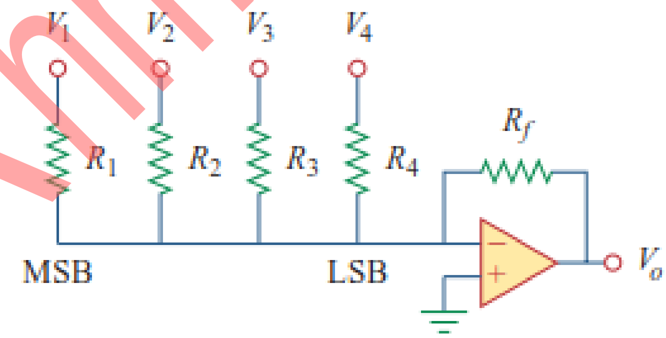
$$-V_0 = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4$$

where: V_1 – MSB (most significant bit), V_4 – LSB (least significant bit);

V_1 to V_4 are either 0 or 1 V.

Example 5.10.

In the op amp circuit of Fig. , let R_f = 10 kΩ, R_1 = 10 kΩ, R_2 = 20 kΩ, R_3 = 40 kΩ, and R_4 = 80 kΩ. obtain the analog output for binary inputs [0000], [0001], [0010], . . . , [1111].



Solution:

Substituting the given values of the input and feedback resistors in following Eq. gives

$$-V_0 = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4 = V_1 + 0.5V_2 + 0.25V_3 + 0.125V_4$$

Using this equation, a digital inputs produce an analog output as following:

$$[V_1V_2V_3V_4] = [0000] \Rightarrow -V_o = 0 \text{ V}$$

$$[V_1V_2V_3V_4] = [0001] \Rightarrow -V_o = 0.125 \text{ V}$$

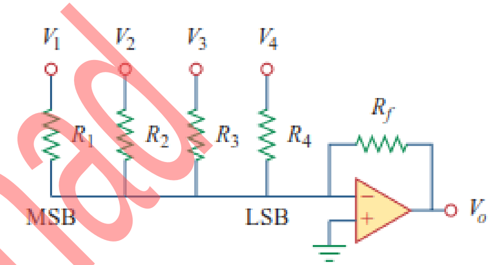
$$[V_1V_2V_3V_4] = [0010] \Rightarrow -V_o = 0.25 \text{ V}$$

$$[V_1V_2V_3V_4] = [0011] \Rightarrow -V_o = 0.25 + 1.125 = 0.375 \text{ V}$$

$$[V_1V_2V_3V_4] = [0100] \Rightarrow -V_o = 0.5 \text{ V}$$

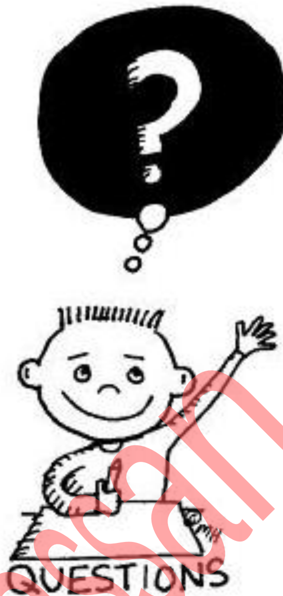
$$\vdots$$

$$[V_1V_2V_3V_4] = [1111] \Rightarrow -V_o = 1 + 0.5 + 0.25 + 0.125 = 1.875 \text{ V}$$



Input and output values of the four-bit DAC.

Binary input $[V_1V_2V_3V_4]$	Decimal value	Output $-V_o$
0000	0	0
0001	1	0.125
0010	2	0.25
0011	3	0.375
0100	4	0.5
0101	5	0.625
0110	6	0.75
0111	7	0.875
1000	8	1.0
1001	9	1.125
1010	10	1.25
1011	11	1.375
1100	12	1.5
1101	13	1.625
1110	14	1.75
1111	15	1.875



The end of chapter 5